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07MCA12

First Semester MCA Degree Examination, June/July 2011
Discrete Mathematics

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions.*

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Define set, subset, equal set and power set, with an example each. **(06 Marks)**

b. Prove that :

i) $(A \cup B)' = A' \cap B'$

ii) $(A \cap B)' = A' \cup B'$

iii) $A \cup (A \cap B) = A$. **(06 Marks)**

c. In a survey of 100 families the numbers that read the most recent issue of various magazines were found to be as follows :

Reader digest	= 28
Reader digest and science today	= 8
Science today	= 30
Reader digest and caravan	= 10
Caravan	= 42
Science today and caravan	= 5
All the magazines	= 3

Find :

i) How many read none of the three magazines?

ii) How many read caravan as their only magazine?

iii) How many read exactly two magazines? **(08 Marks)**

2 a. Prove by induction $1^2 + 3^2 + 5^2 + \dots\dots\dots + (2n+1)^2 = \dfrac{(n+1)(2n+2)(2n+3)}{3}$. **(07 Marks)**

b. i) Construct the truth table for proposition $(p \to q) \wedge (\sim p \to r)$.

ii) Write the inverse, converse and contra positive for "If two integers are equal then their squares are equal". **(07 Marks)**

c. Show that $p \to q$ and $q \to r$ imply $p \to r$. **(06 Marks)**

3 a. Define tautology and contradiction. Show that $(\sim p \vee \sim q) \leftrightarrow \sim(p \wedge q)$ is a tautology and $(\sim p \vee q) \leftrightarrow (p \wedge \sim q)$ is a contradiction. **(06 Marks)**

b. i) Define principal disjunctive normal form and obtain the PDNF of $p \vee (\sim p \vee q)$.

ii) Define principal conjunctive normal form and obtain the PCNF of $(p \wedge q) \vee (\sim p \vee r)$. **(08 Marks)**

c. If $p(x) = x^2 - 7x \le 0$ and $q(x) : (x - 2) < 3$, find the truth set $p(x) \wedge q(x)$. Given that the replacement set being z, the set of all integer. **(06 Marks)**

4 a. Define a relation with an example. If $A = \{1, 2, 3, 4\}$ write the relation of the following :

i) $R_1 = \{(a, b) / a \le b\}\ \forall\ a, b \in A$

ii) $R_2 = \{(a, b) / a = b\}$

iii) $R_3 = \{(a, b) / ab \ge 3\}$

iv) $R_4 = \{(a, b) / a^2 + b^2 \le a\}$. **(06 Marks)**

b. If $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $M_S = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ then find :

i) $M_{R \cup S}$; ii) $M_{R \cap S}$; iii) M_{SOR} ; iv) M_R^{-1}. **(08 Marks)**

c. Let A = {1, 2, 3, 4, 5, 6, 7} and R = {<x, y> / x – y is divisible by 3}, show that 'R' is a equivalence relation. **(06 Marks)**

5 a. Define a partially ordered set. Draw the Hasse diagram representing the positive divisor of 45. **(06 Marks)**

b. Define injective function, bijective function and surjective function. Give one example each. **(06 Marks)**

c. Define composite function, with an example. If f(x) = x + 2, g(x) = x – 2, h(x) = 3x for $x \in R$, where R is the set of real numbers, find : i) fog ; ii) fof ; iii) hof ; iv) fohog. **(08 Marks)**

6 a. Define the following, with an example each :

i) Simple graph and multi-graph.
ii) Directed graph and undirected graph.
iii) Finite graph and infinite graph.
iv) Euler graph and Hamilton graph.
v) Pendent vertex and isolated vertex. **(10 Marks)**

b. Show that :

i) The maximum number of edges in a simple graph with 'n' vertices is n (n – 1) / 2.
ii) Every graph contains even number of odd degree vertices. **(10 Marks)**

7 a. Explain the different operations defined on graphs. **(10 Marks)**

b. Define a tree and spanning tree. If G(p, q) is a connected graph and q = p – 1, prove that G is a tree. **(06 Marks)**

c. Define coloring and chromatic number of a graph. List the properties of chromatic numbers. **(04 Marks)**

8 a. Show that in a group (G, ∗)

i) If a is any element, then $(a^{-1})^{-1} = a$.
ii) The identity element is unique. **(06 Marks)**

b. Prove that the set of rational numbers other than 1 forms an abelian group with respect to the binary operation ∗ defined by a ∗ b = a + b – ab. **(08 Marks)**

c. If H is a normal subgourp of G and K is any subgroup of G, then prove that HK is a subgroup of G. **(06 Marks)**

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Note: ***Answer any FIVE full questions.***

1 a. Define the power set of a set. Find the power set of the set A = { 0, 1, ϕ }. **(04 Marks)**

b. State and prove the De Morgan's laws of set theory. **(05 Marks)**

c. For any three sets A, B and C, if $A \cap B = B \cap C$ and $A \cup C = B \cup C$ then prove that A = B. **(05 Marks)**

d. A computer services company has 300 programmers. It is known that 180 of these can program in PASCAL, 120 in FORTRAN, 30 in C++, 12 in PASCAL and C++, 18 in FORTRAN and C++, 12 in PASCAL and FORTRAN and 6 in all three languages.

i) If a programmer is selected at random, what is the probability that she can program in exactly two languages?

ii) If two programmers are selected at random, what is the probability that they can both program (A) in PASCAL? (B) only in PASCAL? **(06 Marks)**

2 a. Define the tautology and contradiction. Prove that the compound proposition $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology. **(05 Marks)**

b. Prove the logical equivalence $(p \to q) \wedge [\sim q \wedge (r \vee \sim q)] \Leftrightarrow (q \vee p)$ by using the laws of logic. **(05 Marks)**

c. For any propositions p and q, prove the following:
i) $\sim (p \downarrow q) \Leftrightarrow \sim p \uparrow \sim q$ ii) $\sim (p \uparrow q) \Leftrightarrow \sim p \downarrow \sim q$ **(04 Marks)**

d. Test the validity of the following argument:

$p \to q$
$r \to s$
$t \vee \sim s$
$\sim t \vee u$
$\sim u$
$\therefore \sim p$ **(06 Marks)**

3 a. Define an open statement. Write the negation of "If k, m and n are any integers where (k - m) and (m - n) are odd then (k - n) is even". **(06 Marks)**

b. For the universe of all integers, let p(x) : x > 0, q(x) : x is even, r(x) : x is a perfect square, s(x) : x is divisible by 3, t(x) : x is divisible by 7. Write the following statements in symbolic form and indicate their truth value:

i) At least one integer is even.
ii) There exists a positive integer that is even.
iii) If x is even and a perfect square then x is not divisible by 3.
iv) If x is odd or is not divisible by 7 then x is divisible by 3. **(08 Marks)**

c. Give i) A direct proof, ii) An indirect proof and iii) Proof by contradiction, for the following statement: "If n is an odd integer, then n + 9 is an integer". **(06 Marks)**

4 a. Prove by mathematical induction: $1^2 + 3^2 + 5^2 + \ldots\ldots. + (2n-1)^2 = \frac{n(4n^2-1)}{3}$. **(05 Marks)**

b. Prove by induction $n! > 2^n$ for all the integers $n \geq 4$. **(05 Marks)**

c. Obtain a recursive definition for the sequence $\{a_n\}$ in each of the following:
i) $3n + 7$ ii) $n(n+2)$ iii) $2 - (-1)^n$ **(06 Marks)**

d. A sequence $\{a_n\}$ is defined recursively by $a_1 = 4$, $a_n = a_{n-1} + n$ for $n \geq 2$. Find $\{a_n\}$ in explicit form. **(04 Marks)**

5 a. Define the Cartesian product of two sets. For any non-empty sets A, B and C, prove that
$A \times (B - C) = (A \times B) - (A \times C)$ **(05 Marks)**

b. For the sets $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$ and the relations $R = \{(a, 1), (b, 1), (c, 2), (c, 3)\}$ and $S = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$ from A to B. Compute $M_{(R \cup S)}$, $M_{(R \cap S)}$, $M_{(R^c)}$ and $M_{(\bar{S})}$. **(04 Marks)**

c. Let $A = \{1, 2, 3, 4, 5\}$. Define a relation R on A×A by $(x_1, y_1) R (x_2, y_2)$ iff $x_1 + y_1 = x_2 + y_2$. i) Verify that R is an equivalence relation on A×A. ii) Determine the equivalence classes of [(1, 3)], [(2, 4)] and [(1, 1)]. **(06 Marks)**

d. If R is a relation on the set $A = \{1, 2, 3, 4, 6, 12\}$ defined by xRy iff x divides y. Prove that (A, R) is a poset. Draw its Hasse diagram. **(05 Marks)**

6 a. State the Pigeonhole principle. Prove that if 30 dictionaries in a library contain a total of 61,327 pages then at least one of the dictionaries must have at least 2045 pages. **(05 Marks)**

b. Define Stirling number of second kind. Let $A = \{1,2,3,4,5,6,7\}$ and $B = \{w,x,y,z\}$. Find the number of functions and the number of onto functions from A and B. **(05 Marks)**

c. Prove that a function $f : A \rightarrow B$ is invertible iff it is one to one and onto. **(05 Marks)**

d. Define the floor and ceiling functions. Let $A = B = R$. Determine the $\Pi_A(D)$ and $\Pi_B(D)$ for the set $D = \{(x, y) \mid x^2 + y^2 = 1\}$. **(05 Marks)**

7 a. Prove that a group G is abelian iff $(ab)^2 = a^2b^2$ for all $a, b \in G$. **(05 Marks)**

b. Define cyclic group. Prove that every subgroup of a cyclic group is cyclic. **(05 Marks)**

c. State and prove the Lagrange's theorem. **(05 Marks)**

d. The generator matrix for an encoding function $E : Z_2^3 \rightarrow Z_2^3$ is given by $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$

i) Find the code words assigned to 110 and 010.
ii) Obtain the associated parity-check matrix. **(05 Marks)**

8 a. Define group code. Let $E : Z_2^m \rightarrow Z_2^n$, $m < n$ be the encoding function given by a generator matrix G or the associated parity-check matrix H, then prove that $C = E(Z_2^m)$ is a group code. **(10 Marks)**

b. Define a ring and an integral domain. Let R be a commutative ring with unity. Prove that R is an integral domain iff for all $a, b, c \in R$ where $a \neq 0$, $ab = ac \Rightarrow b = c$. **(10 Marks)**

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